# SENTIFIC SECTION

# SECONDARY 2

# PURE MATHIMATICS



# Geel 2000 Language Schools

## Math Department

## First Term

## 2024 / 2025

Name

.......................................................................................

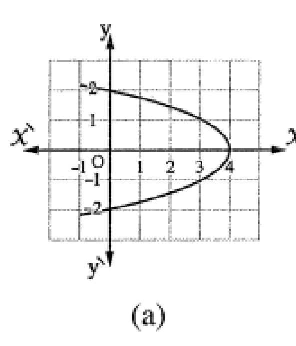
Class.................................................................................

(Unit 1) Real functions

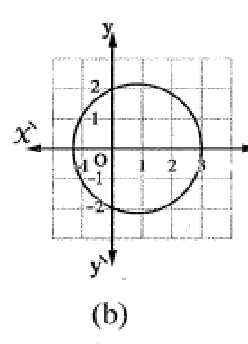
(Domain, Range and monotony)

1-

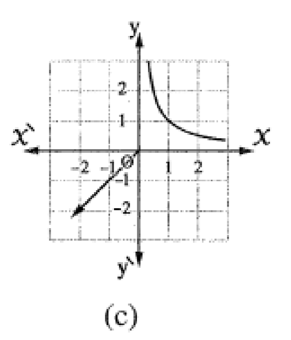
( 1 ) Which of the following figures represents a function of $X$ ?



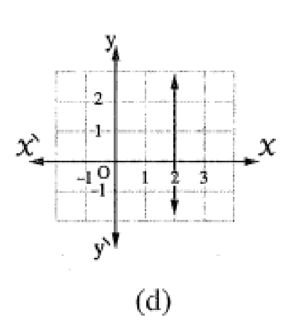

(a)




(b)




(c)




(d)

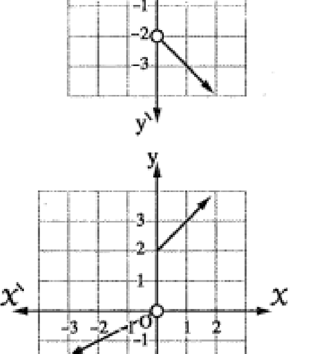
( 2 ) The opposite figure represents a function of $X$ whose domain is ……………

(a) $\mathbb{R}$ (b) $\mathbb{R} - ]-1 , 2[$

(c) $\mathbb{R} - [-1 , 2]$ (d) $\mathbb{R} - \{0\}$

( 3 ) The opposite figure represents a function of $X$ whose range is ……………






(a) $\mathbb{R} - [0 , 2]$ (b) $\mathbb{R} - \{0\}$

(c) $\mathbb{R} - [0 , 2[$ (d) $\mathbb{R} - ]0 , 2]$

( 4 ) $f(X) = \sqrt{4 - X^2}$ , then the domain of the function $f$ = ……………



(a) $[-2 , 2]$ (b) $]-2 , 2[$

(c) $[-2 , 2[$ (d) $]-2 , 2]$

# Operations on functions (composition functions)

2-

**Choose the correct answer from those given :**

( **1** ) $f(x) = \frac{1}{x}$ , $g(x) = \sqrt{x}$ , then the domain of $(f \cdot g) =$ ...............

(a) $\mathbb{R} - \{0\}$ (b) $\mathbb{R}$ (c) $\mathbb{R}^+$ (d) $[0, \infty[$

( **2** ) $f(x) = x + 1$ , $g(x) = x^2$, then $(f \circ g)(2) =$ ...............

(a) 3 (b) 4 (c) 5 (d) 9

( **3** ) The domain of the function $f : f(x) = \sqrt{5 - x}$ equals ...............

(a) $\mathbb{R} - \{5\}$ (b) $\mathbb{R}^+$ (c) $]-\infty, 5]$ (d) $[5, \infty[$

( **4** ) $f(x) = \sqrt{x}$ , $g(x) = x^2$ , then the domain of $(f \circ g) =$ ...............

(a) $[0, \infty[$ (b) $\mathbb{R}$ (c) $\mathbb{R}^+$ (d) $\mathbb{R}^-$

3-

If $f(x) = \frac{1}{x}$ , $g(x) = x + 3$ , find :

( **1** ) $(f \circ g)(x)$

( **2** ) $(g \circ f)(x)$

and state the domain in each case.

4-

If $f(x) = \frac{1}{x}$ , $g(x) = x + 3$ , find :

( **1** ) $(f \circ g)(x)$

( **2** ) $(g \circ f)(x)$

and state the domain in each case.

5-

If $f(\mathcal{X}) = \frac{1}{\mathcal{X}}$ , $g(\mathcal{X}) = 2\mathcal{X}$ , find each of the following :

(1) $(f + g)(\mathcal{X})$ (2) $(f . g)(\mathcal{X})$ (3) $\left(\frac{g}{f}\right)(\mathcal{X})$

Showing the domain.

# Properties of functions

6-

**Find the type of each function whether it is even , odd or otherwise :**

(1) $f(x) = \frac{x^3}{|x| + 2}$ (2) $f(x) = \sin x^2 - \sin^2 x$

7-

[b] If $f(x) = x - 1$ , $g(x) = \sqrt{x}$ , then find $(g \circ f)(x)$ and determine its domain , then find $(g \circ f)(5)$

8-

From the following functions , the one-to-one function is ..............

(a) $f_1(x) = x + 2$ (b) $f_2(x) = x^2$ (c) $f_3(x) = |x|$ (d) $f_4(x) = 5$

9-

Find the domain of $f : f(x) = \frac{3x - 5}{x - 2}$ , then prove that $f$ is one-to-one.

10-

Draw the graph of the function $f : f(x) = \begin{cases} x|x| & \text{when } x < 0 \\ \frac{x^4}{|x|} & \text{when } x > 0 \end{cases}$

, then deduce its domain and discuss its type whether it is even , odd or otherwise.

# Graphical (basic and piecewise)

11-

Graph the function $f : f(x) = 4 - (x - 2)^2$ , then deduce its range , its monotony and whether the function is odd , even or otherwise.

12-

Graph the function $f : [-2 , 6] \longrightarrow \mathbb{R}$ where $f(x) = \begin{cases} 4 - x & , \quad -2 \le x < 1 \\ x & , \quad 1 \le x \le 6 \end{cases}$

and from the graph deduce its range and discuss its monotonicity.

13-

Graph the function $f : f(x) = \begin{cases} x - 1 & , \quad 2 < x \le 4 \\ -1 & , \quad -2 \le x \le 2 \end{cases}$

from the graph determine its range.

14-

**[b]** If $f : [-4 , 3] \longrightarrow \mathbb{R}$ , $f(x) = \begin{cases} 4 & \text{when } x < 0 \\ (x - 1)^2 + 1 & \text{when } 0 \le x \le 3 \end{cases}$

, graph the function $f$ , then from the graph , **deduce :**

( **1** ) The range.

( **2** ) The monotonicity.

( **3** ) The type (even , odd , otherwise).

# Geometric transformations

15-

Use the graph of the function $f$ where $f(X) = X^2$ to represent the function g where $g(X) = (X-1)^2 + 2$ and from the graph determine the range of the function g and discuss its monotonicity and tell whether it is even, odd or otherwise.

16-

Use the curve of the function $f$ where $f(X) = X^3$ to represent each of the following functions :

(1) $f_1(X) = (X+1)^3$ (2) $f_2(X) = X^3 + 1$

17-

If $f(X) = X^2 - 1$ , $g(X) = X + 1$

Graph the function $\frac{f}{g}$ , show its domain and its range.

v

18-

Graph the curve of the function $f : f(X) = X|X|$ , then discuss its monotony and its type whether its even, odd or otherwise.

Solving absolute value equations and inqualities

19-

**Find in $\mathbb{R}$ the solution set for each of the following :**

(1) $|2x-5| \leq 3$

20-

**Find in $\mathbb{R}$ the S.S. of the following equation algebraically :** $\sqrt{x^2-4x+4}=4$

21-

**Find in $\mathbb{R}$ the S.S. of the following inequality algebraically :** $|x-3| \geq 5$

22-

**Find the solution set in $\mathbb{R}$ of each of the following :**

(1) $|2x-3|+|6-4x| \leq 0$

(2) $|2x-4| = x+3$

23-

**Find graphically in $\mathbb{R}$ the solution set of the inequality :** $|5-x| > 3$

24-

**Find in $\mathbb{R}$ the solution set of the inequality :** $\sqrt{4x^2-12x+9} > 5$

25-

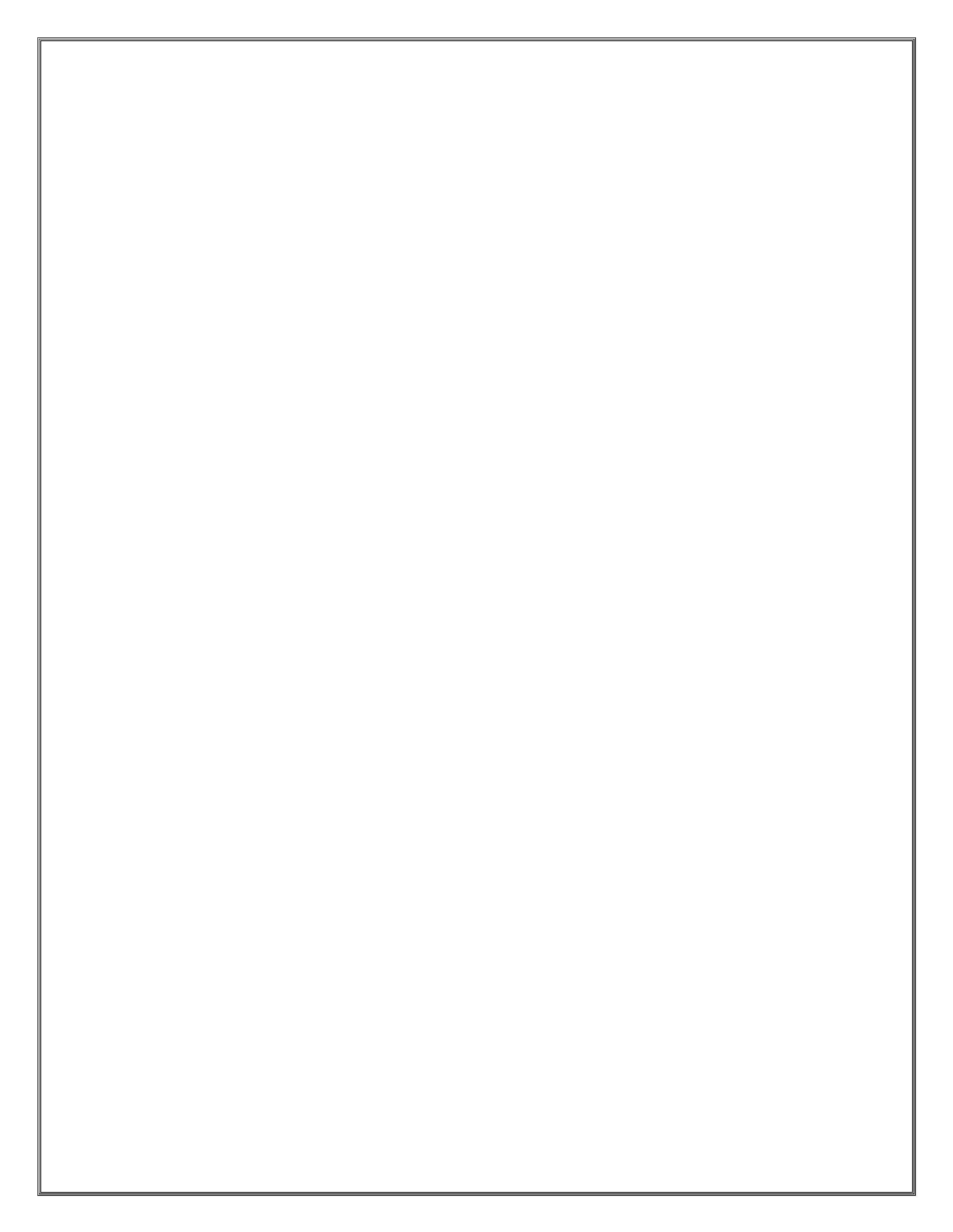
**Find in $\mathbb{R}$ the solution set of each of the following :**

(1) $|x+2|+5=9$

(2) $|2x-3| \leq 5$

26-

**Solve in $\mathbb{R}$ the equation :** $2|x-2|-|2-x|=3$



(Unit 2 )

Rational exponents and exponintial equations

1-

**Find the value of $x$ which satisfies :** $2^{x+1} = 25$ by using calculator.

2-

**Find in $\mathbb{R}$ the solution set of each of the following :**

$$\left(\frac{1}{2}\right)^{x+1} + \left(\frac{1}{2}\right)^{x+3} + \left(\frac{1}{2}\right)^{x+5} = 84$$

3-

**Simplify :** $\dfrac{16^{x+\frac{1}{4}} \times 9^{x+2}}{8^{x-1} \times 18^{x+2}}$

4-

**Find the S.S. of each of the following in $\mathbb{R}$ :**

$$4^{x} + 2^{x+1} = 8$$

5-

**Put in the simplest form :** $\dfrac{9^{4n+1} \times 4^{2-2n}}{3^{9n+1} \times 48^{1-n}}$

6-

If $f(2x) = 3^{x}$ **, solve in $\mathbb{R}$ the equation :** $f(2x+4) + f(2x) = 90$

# Applications

7-

A patient gets 40 mg. of medicine , his body gets rid of 10 % of this medicine every hour.

( 1 ) Write the exponential function which represents the quantity of medicine left after t hours.

( 2 ) Estimate this quantity of medicine left in the body after 4 hours.

8-

The number of cows in a cattle farm is 80 cows and the reproduction rate of these cows is 18 % annually. Find the number of cows after 4 years , given by $C = a\,(1 + r)^t$ where t is the number of years , a is the starting amount and r is the annual percentage of reproduction.

The invers function

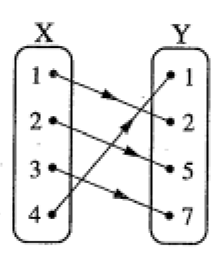
9-

The opposite figure represents a function $f : X \longrightarrow Y$

, then $f^{-1}(2) =$ ...............

(a) 5 (b) 1

(c) 3 (d) 4




10-

If $f(X) = 3 + \sqrt{2X - 1}$ , **find :** $f^{-1}(X)$ and find the domain of $f^{-1}$ and its range.

11-

If $f(X) = X - 3$ , then find the inverse function of $f$

12-

Find the inverse function of $f$ where $f(X) = X^3 + 1$

13-

If $f(X) = 5X$ , find $f^{-1}(X)$ and represent it graphically.

## Logarithmic function and its properties

14-

**Without using calculator , find the value of :** $\log 25 + \frac{\log 8 \times \log 16}{\log 64}$

15-

| **Find in $\mathbb{R}$ the S.S of the equation :**

$\log_7 \mathcal{X} + \log_7 (\mathcal{X} + 6) = 1$

16-

If $\mathcal{X}$ y = 16 **, prove that :** $3 \log_2 \mathcal{X} + 4 \log_2 y - \log_2 \mathcal{X} y^2 = 8$

17-

**Find in $\mathbb{R}$ the solution set of each of the following :**

$\log_5 (\mathcal{X}^2 - 25) - \log_5 (\mathcal{X} - 5) = 2$

18-

**Find the S.S. in $\mathbb{R}$ of the equation :**

$(\log_3 \mathcal{X})^2 - 2 \log_3 \mathcal{X} - 3 = 0$

19-

**Simplify : ( 1 )** $\frac{1}{\log_a ab} + \frac{1}{\log_b ab}$

20-

**Find the S.S. of the following equation in $\mathbb{R}$ :** $\log_2 \mathcal{X} + \log_2 (\mathcal{X} + 1) = 1$

21-

**Without using calculator , find the value of the following :**

$2 \log 25 + \log \left(\frac{1}{3} + \frac{1}{5}\right) + 2 \log 3 - \log 30$

22-

**Solve the equation in $\mathbb{R}$ :** $\log_2 x + \log_2 (x + 1) = 1$

23-

**Solve in $\mathbb{R}$ the following equation :** $\log_4 x = 1 - \log_4 (x - 3)$

24-

**Find in $\mathbb{R}$ the solution set for each of the following :**

$\log_3 x + \log_x 3 = 2$

25-

**( 1 ) Find in $\mathbb{R}$ the solution set of the equation :** $\log_3 x + \log_x 3 = 2$

**( 2 ) Prove that :** $\frac{\log 729 - \log 64}{\log 9 - \log 4} = 3$

26-

**Prove that :** $\log_2 \frac{4}{11} - \log_2 \frac{7}{130} + \log_2 \frac{77}{65} = \log_5 125$

27-

**Find in $\mathbb{R}$ the S.S. of the equation :**

$\log_4 x = 1 - \log_4 (x - 3)$

28-

**Prove that :** $\log_b a \times \log_c b \times \log_d c \times \log_a d = 1$

# (Unit 3 ) Introduction of limits of functions

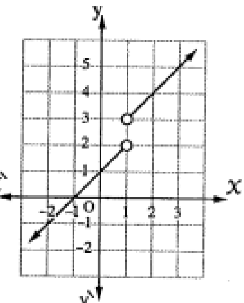
1-

**Choose the correct answer from those given :**

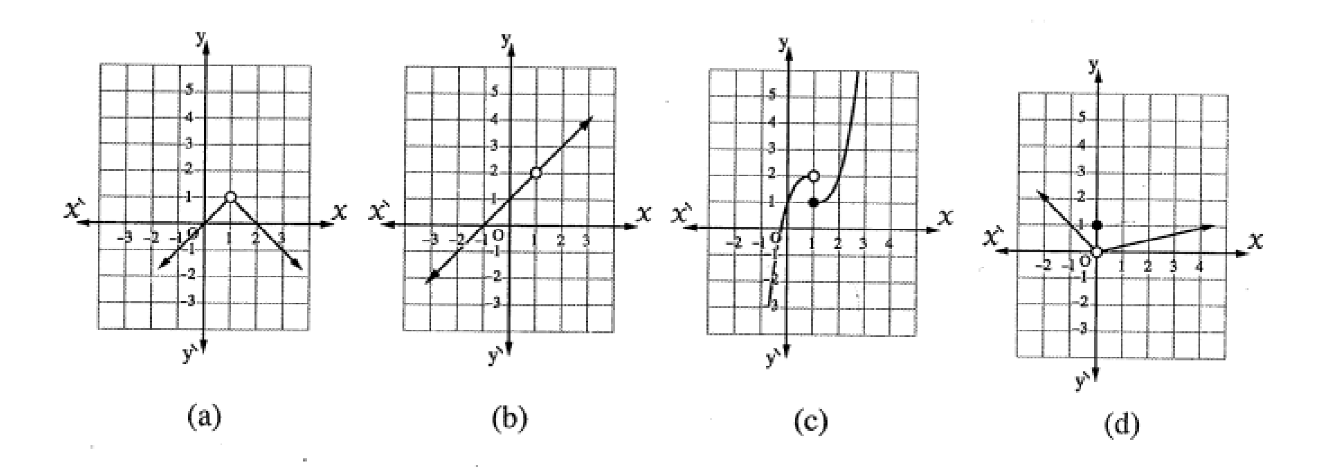
**( 1 )** The opposite figure represents the graph of the function $f$ , then

$\lim\limits_{x \to 1} f(x) =$ ...............

(a) 2  (b) 3  (c) 1  (d) not exist



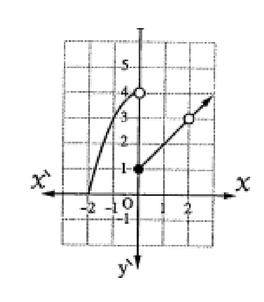
**( 2 )** Which of the following functions has no limit at $x = 1$ ?



(a)  (b)  (c)  (d)

2-

**In the opposite figure , find :**



**( 1 )** $f(\text{zero}^+)$  **( 2 )** $f(\text{zero}^-)$

**( 3 )** $f(2)$  **( 4 )** $\lim\limits_{x \to 2} f(x)$

## Finding tke limit of the function algebrically

3-

**Choose the correct answer from those given :**

(1) $\lim\limits_{x \to 0} \frac{1+x}{4x-1} = \ldots\ldots\ldots\ldots\ldots$

(a) $-1$ (b) $\frac{1}{4}$ (c) $-\frac{1}{4}$ (d) $1$

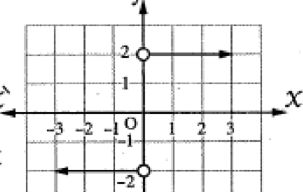
(2) $\lim\limits_{x \to 3} \frac{x^2-9}{x-3} = \ldots\ldots\ldots\ldots\ldots$

(a) $-6$ (b) zero (c) $3$ (d) $6$

(3) The opposite figure represents $f(x)$

Then : $\lim\limits_{x \to 2} f(x) = \ldots\ldots\ldots\ldots\ldots$

(a) $0$ (b) $-2$ (c) $2$ (d) not exist




4-

**Find :**

$$\lim_{x \to 3} \frac{x^2-x-6}{x-3}$$

5-

**Find :**

$$\lim_{x \to -1} \left(\frac{5x^2+5x}{3x^2-3}\right)$$

6-

$$\lim_{x \to 0} \frac{x^2+x}{x} = \ldots\ldots\ldots\ldots$$

7-

**Find :** $\lim\limits_{x \to 1} \frac{x^3-2x^2+1}{x^2+x-2}$

8-

**Find :** $(2)\ \lim_{x \to 2} \frac{x^2 - 5x + 6}{x^3 - 8}$

9-

**Find :** $\lim_{x \to 5} \frac{x - 5}{2x - 3}$

10-

**Find :** $\lim_{x \to 2} \frac{x^3 + 8}{x + 2}$

11-

$\lim_{x \to -3} \frac{x^2 + 4x + 3}{x^2 - 9}$

12-

**Find :** $(1)\ \lim_{x \to -2} \frac{x^3 + 8}{x + 2}$

The law

13-

$$\lim_{x \to 4} \frac{\sqrt{x-3}-1}{x-4}$$

14-

**Find : ( 1 )** $\lim_{x \to -2} \frac{(x+3)^5-1}{x+2}$

15-

**Find : ( 1 )** $\lim_{x \to 2} \frac{x^3+8}{x+2}$

16-

| **Find : ( 1 )** $\lim_{x \to 3} \frac{x^5-243}{x^2-9}$

17-

**Find :** $\lim_{x \to 5} \frac{(x-3)^5-32}{x-5}$

18-

**Find : ( 1 )** $\lim_{x \to -3} \frac{x^4-81}{x^5+243}$

19-

**Find : ( 1 )** $\lim_{x \to -2} \frac{\sqrt[3]{x} + \sqrt[3]{2}}{x+2}$

20-

**Find : ( 1 )** $\lim_{x \to \sqrt{5}} \frac{x^7 - 125\sqrt{5}}{x^4 - 25}$

21-

$\lim_{x \to 5} \frac{(x-3)^5 - 32}{x^2 - 5x}$

22-

$\lim_{x \to -3} \frac{x^4 - 81}{x^5 + 243}$

23-

$\lim_{x \to -3} \frac{x^4 - 81}{x^5 + 243}$

24-

$\lim_{x \to 2} \frac{x^7 - 128}{x^5 - 32}$

25-

**Find : ( 1 )** $\lim_{x \to 2} \frac{2x^4 - 32}{x^2 - 4}$

26-

**Find : ( 1 )** $\lim\limits_{x \longrightarrow \frac{1}{3}} \frac{27x^4 - \frac{1}{3}}{3x - 1}$

Limit of the function at infinity

26-

**Find : ( 1 )** $\lim_{x \to \infty} \frac{2x^3 - 5x}{x^4 + 3}$

27-

$\lim_{x \to \infty} \frac{2x^3 - 9}{|3x|^3 + 7}$

28-

$\lim_{x \to \infty} \frac{5x^{-3} + 4x^{-2} - 3}{7x^{-3} - 2x^{-2} + 8}$

29-

**Find : ( 1 )** $\lim_{x \to \infty} \left(\sqrt{4x^2 - 2x + 1} - 2x\right)$

30-

$\lim_{x \to \infty} \frac{2x^2 - x + 1}{x^3 - x^2 + 1}$

31-

**Find : ( 1 )** $\lim_{x \to \infty} \frac{2x^3 - 3}{3x^2 + 1}$

32-

$$\lim_{x \to \infty} \frac{x^{-2} + 3}{x^{-3} + 6}$$

33-

**Find : ( 1 )** $\lim_{x \to \infty} \frac{\sqrt{9x^2 + 3}}{6x - 1}$

34-

$$\lim_{x \to \infty} \frac{5 - 6x - 3x^2}{2x^2 + x + 4}$$

35-

$$\lim_{x \to \infty} \frac{-x}{\sqrt{4 + x^2}}$$

36-

**Find : ( 1 )** $\lim_{x \to \infty} \frac{x^3 - 4x + 5}{(2x - 1)^3}$ .

37-

**Find : ( 1 )** $\lim_{x \to \infty} \frac{5 - 6x - 3x^2}{2x^2 + x + 4}$

38-

**Find : ( 1 )** $\lim_{x \to \infty} \frac{3x^2 + x - 1}{8x^2 - 3}$

39-

$$\lim_{x \to \infty} \frac{2x^{-1} - 3x^{-2}}{4 + x^{-1}}$$

40-

$$\lim_{x \to \infty} \frac{3x + \sqrt{4x^2 + 5}}{5x - 3}$$

41-

$$\lim_{x \to \infty} \left(\sqrt{x^2 + 5x} - x\right)$$

42-

$$\lim_{x \to \infty} \frac{x^3 - 2}{|x|^3 + 1}$$

43-

**Find : ( 1 )** $\lim_{x \to \infty} \frac{x^3 - 4x + 5}{(2x - 1)^3}$

44-

**Find :** ( 1 ) $\lim_{x \to \infty} (\sqrt{x^2 + 5x} - x)$

45-

$\lim_{x \to \infty} \frac{1}{x}\sqrt{3 + 4x^2}$

Limits of tragnometric functions

46-

$$\lim_{x \to 0} \frac{1-\cos^2 2x}{3x^2}$$

47-

$$\lim_{x \to 0} \frac{x - x\cos x}{\sin^2 3x}$$

48-

$$\lim_{x \to 0} \frac{\sin 3x - \sin 2x}{5x}$$

49-

**Find : ( 1 )** $\lim_{x \to 0} \frac{2x + \sin 3x}{\tan 5x}$

50-

**Find :** $\lim_{x \to 0} \frac{x^3 + \sin 3x}{5x\cos 2x}$

51-

**Find : ( 1 )** $\lim_{x \to 0} \frac{x - x\cos x}{\sin^2 3x}$

52-

$\lim_{x \to 0} \frac{x^2 + \sin 3x}{5x\cos 2x}$

53-

$\lim_{x \to 0} \frac{\sin^2 3x - \tan 2x^2}{x^2 \cos 4x}$

54-

$\lim_{x \to 0} \frac{x\tan 2x\cos 3x}{x^2 + \sin^2 3x}$

55-

**Find : ( 1 )** $\lim_{x \to 1} \frac{\sin(x - 1)}{x^2 + x - 2}$

56-

**Find : ( 1 )** $\lim_{x \to 0} \frac{x - x\cos x}{\sin^2 3x}$

57-

**Find : ( 1 )** $\lim_{x \to 0} \frac{1-\cos x+\sin x}{1-\cos x-\sin x}$

58-

$$\lim_{x \to 0} \frac{\sin 3x-\tan 2x}{5x}$$

59-

$$\lim_{x \to 0} \frac{\sin 2x+5\sin 3x}{x}$$

60-

$$\lim_{x \to 0} \frac{x\tan x+\sin^2 3x}{2x^2+\sin 3x^2}$$

Existence of the limit of a piecewise function

61-

**Find :** (1) $\lim\limits_{x \longrightarrow \pi} f(x)$ where $f(x) = \begin{cases} \dfrac{2 \sin x}{\pi - x} & , \quad x < \pi \\ 1 - \cos x & , \quad x > \pi \end{cases}$

(2) $\lim\limits_{x \longrightarrow \infty} x \tan x^{-1}$

62-

$$f(x) = \begin{cases} \dfrac{x \tan x + \sin^2 3x}{5x^2} \\ 2 \cos 2x \end{cases}$$

63-

Discuss the existence of the limit of the function $f$ where

$$f(x) = \begin{cases} \dfrac{1 - \cos x}{x} & , \quad x > 0 \\ 2 \sin x & , \quad x \leq 0 \end{cases} \text{ at } x = 0$$

64-

**Find :** $\lim\limits_{x \to 3} f(x)$ if $f(x) = \begin{cases} x^2 + 1 & , \quad x < 3 \\ 3x + 1 & , \quad x > 3 \end{cases}$

65-

| If $f(x) = \begin{cases} x^2 - 2x & , \quad -2 < x < 1 \\ 3x - 4 & , \quad 1 \le x < 4 \end{cases}$

**, discuss the existence of each of the following :**

**(1)** $\lim\limits_{x \to -2} f(x)$ **(2)** $\lim\limits_{x \to 1} f(x)$ .

66-

If $\lim\limits_{x \to 2} f(x) = 7$ where $f(x) = \begin{cases} x^2 + 3m & , \quad x < 2 \\ 5x + k & , \quad x > 2 \end{cases}$ , find m and k

67-

Discuss the existence of Lim $f(x)$ which $x \longrightarrow 0$ where

$$f(x) = \begin{cases} \dfrac{5x^2 + \tan 2x^2}{\sin^2 x} & , \quad x > 0 \\ 7 \cos 3x & , \quad x < 0 \end{cases}$$

68-

**Find :** (1) $\lim_{x \to \pi} f(x)$ where $f(x) = \begin{cases} \dfrac{2 \sin x}{\pi - x} & , \quad x < \pi \\ 1 - \cos x & , \quad x > \pi \end{cases}$

(2) $\lim_{x \to \infty} x \tan x^{-1}$

69-

If $f(x) = \begin{cases} \dfrac{x^2 - 7x + 12}{x - 3} & , \quad x > 3 \\ a x - 7 & , \quad x < 3 \end{cases}$

, $\lim_{x \to 3} f(x) = -1$ , then find the value of a

Continuity

70-

Discuss the continuity of the function $f$

where $f(x) = \begin{cases} 8 - x & , \quad x \geq 3 \\ x + 2 & , \quad x > 3 \end{cases}$ at $x = 3$

71-

**Discuss the continuity of the function $f$ where :**

$$f(x) = \begin{cases} \dfrac{x^2 + x - 2}{x + 2} & , \quad x > -2 \\ 3x + 5 & , \quad x \leq -2 \end{cases} \text{ at } x = -2$$

72-

**Discuss the continuity of the function $f$ on its domain where :**

$$f(x) = \begin{cases} 1 + \sin x & , \quad \text{when } 0 \leq x \leq \dfrac{\pi}{2} \\ 1 - \cos 2x & , \quad \text{when } x > \dfrac{\pi}{2} \end{cases}$$

73-

**Discuss the continuity of the function $f$ :**

$$f(x) = \begin{cases} \dfrac{\sin(x-2)}{x^2-4} & , \quad x < 2 \\ 1 - \dfrac{3}{x^2} & , \quad x \geq 2 \end{cases} \quad \text{at } x = 2$$

74-

Discuss the continuity of $f$ : $f(x) = \begin{cases} \dfrac{x^2 - x - 6}{x - 3} & , \quad x < 3 \\ 2x - 1 & , \quad x \geq 3 \end{cases}$ at $x = 3$

75-

**Discuss the continuity of the function $f$ where**

$$f(x) = \begin{cases} \dfrac{\sin x}{2x - 2\pi} & , \quad x < \pi \\ \dfrac{1}{2} \cos x & , \quad x \geq \pi \end{cases} \quad \text{at } x = \pi$$

76-

**Discuss the continuity of $f$ at $x = 1$**

where $f(x) = \begin{cases} x^2 + 3 & , \quad x \geq 1 \\ \dfrac{x^2 + 2x - 3}{x - 1} & , \quad x < 1 \end{cases}$

77-

**Find the value of a which makes the function $f$**

where $f(\mathcal{X}) = \begin{cases} \dfrac{\mathcal{X}^2 - 5\mathcal{X} + 6}{\mathcal{X}^3 - 8} & , \quad \mathcal{X} \neq 2 \\ \dfrac{-2}{|a|} & , \quad \mathcal{X} = 2 \end{cases}$ continuous at $\mathcal{X} = 2$

78-

Find the value of k which makes the function $f$ continuous at $\mathcal{X} = 2$ where

$$f(\mathcal{X}) = \begin{cases} \dfrac{\mathcal{X}^2 + \mathcal{X} - 6}{\mathcal{X}^3 - 8} & , \quad \mathcal{X} \neq 2 \\ \dfrac{2}{|k|} & , \quad \mathcal{X} = 2 \end{cases}$$

79-

**Discuss the continuity of $f$ :** $f(\mathcal{X}) = \begin{cases} \mathcal{X}^2 + 3 & , \quad \mathcal{X} \geq 1 \\ \dfrac{\mathcal{X}^2 + 2\mathcal{X} - 3}{\mathcal{X} - 1} & , \quad \mathcal{X} < 1 \end{cases}$ at $\mathcal{X} = 1$

80-

**Discuss the continuity of the function $f$**

where $f(\mathcal{X}) = \begin{cases} \mathcal{X}^2 + 3 & , \quad \mathcal{X} \geq 1 \\ \dfrac{\mathcal{X}^2 + 2\mathcal{X} - 3}{\mathcal{X} - 1} & , \quad \mathcal{X} < 1 \end{cases}$ at $\mathcal{X} = 1$

81-

**Find the value of the constant a if the function $f$ :**

where $f(x) = \begin{cases} \dfrac{(x+3)^4 - 81}{x} & , \quad x \neq 3 \\ a & , \quad x = 3 \end{cases}$ is continuous at $x = 3$

82-

**Discuss the continuity of the function $f$ where :**

$f(x) = \begin{cases} \dfrac{\sin x}{\pi - x} & , \quad x \neq \pi \\ 1 & , \quad x = \pi \end{cases}$ at $x = \pi$

## (Unit 4) The sine rule

1-

Solve $\Delta$ ABC in which m ($\angle$ B) = $35^\circ$ , m ($\angle$ C) = $70^\circ$ , and the diameter length of its circumcircle = 32 cm.

2-

In $\Delta$ ABC , if m ($\angle$ A) = $35^\circ$ , a = 17 cm. and b = 20 cm.

**Prove that :** $\Delta$ ABC has two solutions , then find them.

3-

Find the perimeter of $\Delta$ ABC in which m ($\angle$ A) = $57^\circ\ 13'$ , c = 8.7 cm.

and m ($\angle$ B) = $64^\circ\ 18'$

4-

ABC is a triangle in which : m (∠ A) = 35° , a = 8 cm. and b = 6 cm. **Find :** m (∠ B)

5-

ABC is a triangle in which : b = 12 cm. , m (∠ B) = 75° and m (∠ C) = 45° **Find :**

(**1**) a

(**2**) The area of Δ ABC

(**3**) The radius length of the circumcircle of the triangle ABC

6-

ABC is a triangle in which m (∠ A) : m (∠ B) : m (∠ C) = 3 : 4 : 3

If a = 5 cm. , find the perimeter of Δ ABC

7-

Solve the triangle ABC in which a = 8 cm. , m (∠ A) = 60° and m (∠ B) = 40°

8-

Solve the acute-angled triangle ABC in which a = 21 cm. , b = 25 cm. and the diameter length of its circumcircle = 28 cm.

9-

Find the shortest side length in Δ ABC , in which : m (∠ A) = 43° , m (∠ B) = 70° and c = 9 cm. Find the area of the triangle ABC.

10-

ABC is a triangle in which : AC = 4.7 cm. , m (∠ B) = 34° and m (∠ C) = 66°

Find the length of $\overline{BC}$ , then find the area of its circumcircle.

11-

ABC is a triangle in which m (∠ A) = 40° , a = 5 cm. and b = 7 cm.

Find m (∠ B) approximating to the nearest minute.

12-

Solve the triangle ABC in which a = 5 cm. , b = 7 cm. and m (∠ C) = 65°

13-

ABC is a triangle in which m (∠ A) = 85° , m (∠ B) = 55° and c = 5 cm.

Find the area of the circumcircle of Δ ABC

## The cosine rule

14-

The perimeter of the triangle ABC is 52 cm. , a = 13 cm. and b = 17 cm.

Find the measure of the greatest angle.

15-

Solve the triangle ABC in which a = 5 cm. , b = 7 cm. and m (∠ C) = 65°

16-

Solve the triangle ABC in which a = 9 cm. , b = 7 cm. and c = 5 cm. , then find its area.

17-

In $\Delta$ ABC , if $a = 4$ cm. , $b = 5$ cm. and $c = 6$ cm. , **prove that :** $\cos C = \cos 2A$

, then find the circumference of the circumcircle of $\Delta$ ABC

18-

Solve the triangle ABC in which $a = 9$ cm. , $b = 15$ cm. and $m(\angle C) = 106°$

19-

Solve the triangle ABC in which $a = 15$ cm. , $b = 13$ cm. and $c = 14$ cm.

20-

ABC is a triangle in which $a = 27$ cm. , $b = 35$ cm. and $c = 18$ cm.

Find the measure of the greatest angle.

## Solution of the triangle

21-

In $\Delta$ ABC , $\cos A = \frac{2}{5}$ , b = 2.5 cm. and c = 2 cm.

**Prove that :** $\Delta$ ABC is an isosceles triangle and find its area.

22-

In $\Delta$ ABC , if m ($\angle$ A) = 35° , a = 17 cm. and b = 20 cm.

**Prove that :** $\Delta$ ABC has two solutions , then find them.

23-

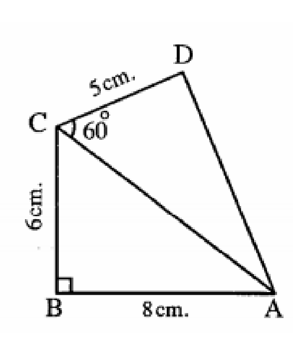
**In the opposite figure :**

ABCD is a quadrilateral in which

AB = 8 cm. , BC = 6 cm. , m ($\angle$ B) = 90°

, DC = 5 cm. and m ($\angle$ ACD) = 60°

Find the area of the circumcircle of $\Delta$ ADC



24-

In $\Delta$ ABC , show whether it has only 1 solution , 2 solutions or non , given your answer to the nearest decimal :

(1) $m(\angle B) = 110°$ , $b = 8$ cm. and $c = 5$ cm.

(2) $m(\angle A) = 60°$ , $a = 7$ cm. and $b = 9$ cm.

25-

Show whether the triangle ABC has one , two or no solution ,

given that : $m(\angle A) = 100°$ , $a = 12$ cm. and $b = 15$ cm.

26-

ABC is a triangle in which $m(\angle A) = 52°$ , $a = 21$ cm. and $b = 26$ cm.

**Prove that :** $\Delta$ ABC has two solutions , then find them.

27-

ABC is a triangle in which : $\frac{\sin A}{3} = \frac{\sin B}{4} = \frac{\sin C}{5}$

and its perimeter = 24 cm. Find its area.